EGMO VIRTUAL
Netherlands | 2020
European Girls' Mathematical Olympiad

Language: **English**

Day: **2**

*April 2020*

**Problem 4.** A permutation of the integers 1, 2, …, $m$ is called *fresh* if there exists no positive integer $k < m$ such that the first $k$ numbers in the permutation are 1, 2, …, $k$ in some order. Let $f_m$ be the number of fresh permutations of the integers 1, 2, …, $m$.

Prove that $f_n \geq n \cdot f_{n-1}$ for all $n \geq 3$.

*For example, if $m = 4$, then the permutation $(3, 1, 4, 2)$ is fresh, whereas the permutation $(2, 3, 1, 4)$ is not.*

**Problem 5.** Consider the triangle $ABC$ with $\angle BCA > 90°$. The circumcircle $\Gamma$ of $ABC$ has radius $R$. There is a point $P$ in the interior of the line segment $AB$ such that $PB = PC$ and the length of $PA$ is $R$. The perpendicular bisector of $PB$ intersects $\Gamma$ at the points $D$ and $E$.

Prove that $P$ is the incentre of triangle $CDE$.

**Problem 6.** Let $m > 1$ be an integer. A sequence $a_1, a_2, a_3, \ldots$ is defined by $a_1 = a_2 = 1$, $a_3 = 4$, and for all $n \geq 4$,

$$a_n = m(a_{n-1} + a_{n-2}) - a_{n-3}.$$

Determine all integers $m$ such that every term of the sequence is a square.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

**To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Saturday 18 April, 22:00 UTC (15:00 Pacific Daylight Time, 23:00 British Summer Time, 08:00 (Sunday) Australian Eastern Standard Time).**