Problem 1. The positive integers $a_0, a_1, a_2, \ldots, a_{3030}$ satisfy
\[ 2a_{n+2} = a_{n+1} + 4a_n \quad \text{for } n = 0, 1, 2, \ldots, 3028. \]
Prove that at least one of the numbers $a_0, a_1, a_2, \ldots, a_{3030}$ is divisible by $2^{2020}$.

Problem 2. Find all lists $(x_1, x_2, \ldots, x_{2020})$ of non-negative real numbers such that the following three conditions are all satisfied:
(i) $x_1 \leq x_2 \leq \ldots \leq x_{2020}$;
(ii) $x_{2020} \leq x_1 + 1$;  
(iii) there is a permutation $(y_1, y_2, \ldots, y_{2020})$ of $(x_1, x_2, \ldots, x_{2020})$ such that
\[ \sum_{i=1}^{2020} ((x_i + 1)(y_i + 1))^2 = 8 \sum_{i=1}^{2020} x_i^3. \]

A permutation of a list is a list of the same length, with the same entries, but the entries are allowed to be in any order. For example, $(2, 1, 2)$ is a permutation of $(1, 2, 2)$, and they are both permutations of $(2, 2, 1)$. Note that any list is a permutation of itself.

Problem 3. Let $ABCDEF$ be a convex hexagon such that $\angle A = \angle C = \angle E$ and $\angle B = \angle D = \angle F$ and the (interior) angle bisectors of $\angle A$, $\angle C$, and $\angle E$ are concurrent.
Prove that the (interior) angle bisectors of $\angle B$, $\angle D$, and $\angle F$ must also be concurrent.

Note that $\angle A = \angle FAB$. The other interior angles of the hexagon are similarly described.