EGMO VIRTUAL
Netherlands | 2020
European Girls' Mathematical Olympiad

Language: **English**

Day: **1**

April 2020

Problem 1. The positive integers $a_0, a_1, a_2, \ldots, a_{3030}$ satisfy

$$2a_{n+2} = a_{n+1} + 4a_n \quad \text{for } n = 0, 1, 2, \ldots, 3028.$$

Prove that at least one of the numbers $a_0, a_1, a_2, \ldots, a_{3030}$ is divisible by 2^{2020}.

Problem 2. Find all lists $(x_1, x_2, \ldots, x_{2020})$ of non-negative real numbers such that the following three conditions are all satisfied:

(i) $x_1 \le x_2 \le \ldots \le x_{2020}$;

(ii) $x_{2020} \le x_1 + 1$;

(iii) there is a permutation $(y_1, y_2, \ldots, y_{2020})$ of $(x_1, x_2, \ldots, x_{2020})$ such that

$$\sum_{i=1}^{2020} \big((x_i + 1)(y_i + 1)\big)^2 = 8 \sum_{i=1}^{2020} x_i^3.$$

A permutation of a list is a list of the same length, with the same entries, but the entries are allowed to be in any order. For example, $(2, 1, 2)$ is a permutation of $(1, 2, 2)$, and they are both permutations of $(2, 2, 1)$. Note that any list is a permutation of itself.

Problem 3. Let $ABCDEF$ be a convex hexagon such that $\angle A = \angle C = \angle E$ and $\angle B = \angle D = \angle F$ and the (interior) angle bisectors of $\angle A$, $\angle C$, and $\angle E$ are concurrent.

Prove that the (interior) angle bisectors of $\angle B$, $\angle D$, and $\angle F$ must also be concurrent.

Note that $\angle A = \angle FAB$. The other interior angles of the hexagon are similarly described.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Saturday 18 April, 22:00 UTC (15:00 Pacific Daylight Time, 23:00 British Summer Time, 08:00 (Sunday) Australian Eastern Standard Time).