Problem 4. A domino is a $1 \times 2$ or $2 \times 1$ tile.

Let $n \geq 3$ be an integer. Dominoes are placed on an $n \times n$ board in such a way that each domino covers exactly two cells of the board, and dominoes do not overlap.

The value of a row or column is the number of dominoes that cover at least one cell of this row or column. The configuration is called balanced if there exists some $k \geq 1$ such that each row and each column has a value of $k$.

Prove that a balanced configuration exists for every $n \geq 3$, and find the minimum number of dominoes needed in such a configuration.

Problem 5. Let $\Gamma$ be the circumcircle of triangle $ABC$. A circle $\Omega$ is tangent to the line segment $AB$ and is tangent to $\Gamma$ at a point lying on the same side of the line $AB$ as $C$. The angle bisector of $\angle BCA$ intersects $\Omega$ at two different points $P$ and $Q$.

Prove that $\angle ABP = \angle QBC$.

Problem 6.

(a) Prove that for every real number $t$ such that $0 < t < \frac{1}{2}$ there exists a positive integer $n$ with the following property: for every set $S$ of $n$ positive integers there exist two different elements $x$ and $y$ of $S$, and a non-negative integer $m$ (i.e. $m \geq 0$), such that

$$|x - my| \leq ty.$$

(b) Determine whether for every real number $t$ such that $0 < t < \frac{1}{2}$ there exists an infinite set $S$ of positive integers such that

$$|x - my| > ty$$

for every pair of different elements $x$ and $y$ of $S$ and every positive integer $m$ (i.e. $m > 0$).