EGMO 2017 Zürich

Language: **English (US)**

Day: **1**

Saturday, April 8, 2017

Problem 1. Let $ABCD$ be a convex quadrilateral with $\angle DAB = \angle BCD = 90^\circ$ and $\angle ABC > \angle CDA$. Let Q and R be points on segments BC and CD, respectively, such that line QR intersects lines AB and AD at points P and S, respectively. It is given that $PQ = RS$. Let the midpoint of BD be M and the midpoint of QR be N. Prove that the points M, N, A and C lie on a circle.

Problem 2. Find the smallest positive integer k for which there exist a colouring of the positive integers $\mathbb{Z}_{>0}$ with k colours and a function $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ with the following two properties:

(i) For all positive integers m, n of the same colour, $f(m + n) = f(m) + f(n)$.

(ii) There are positive integers m, n such that $f(m + n) \neq f(m) + f(n)$.

In a colouring of $\mathbb{Z}_{>0}$ with k colours, every integer is coloured in exactly one of the k colours. In both (i) and (ii) the positive integers m, n are not necessarily different.

Problem 3. There are 2017 lines in the plane such that no three of them go through the same point. Turbo the snail sits on a point on exactly one of the lines and starts sliding along the lines in the following fashion: she moves on a given line until she reaches an intersection of two lines. At the intersection, she follows her journey on the other line turning left or right, alternating her choice at each intersection point she reaches. She can only change direction at an intersection point. Can there exist a line segment through which she passes in both directions during her journey?

Time: 4 hours and 30 minutes
Each problem is worth 7 points