Language: **English**

Day: **2**

Wednesday, April 13, 2016

Problem 4. Two circles, ω_1 and ω_2, of equal radius intersect at different points X_1 and X_2. Consider a circle ω externally tangent to ω_1 at a point T_1, and internally tangent to ω_2 at a point T_2. Prove that lines X_1T_1 and X_2T_2 intersect at a point lying on ω.

Problem 5. Let k and n be integers such that $k \geq 2$ and $k \leq n \leq 2k-1$. Place rectangular tiles, each of size $1 \times k$ or $k \times 1$, on an $n \times n$ chessboard so that each tile covers exactly k cells, and no two tiles overlap. Do this until no further tile can be placed in this way. For each such k and n, determine the minimum number of tiles that such an arrangement may contain.

Problem 6. Let S be the set of all positive integers n such that n^4 has a divisor in the range n^2+1, n^2+2, $\ldots$, n^2+2n. Prove that there are infinitely many elements of S of each of the forms $7m$, $7m+1$, $7m+2$, $7m+5$, $7m+6$ and no elements of S of the form $7m+3$ or $7m+4$, where m is an integer.

Time: 4 hours and 30 minutes
Each problem is worth 7 points