Tuesday, April 12, 2016

Problem 1. Let n be an odd positive integer, and let x_1, x_2, …, x_n be non-negative real numbers. Show that

$$\min_{i=1,\ldots,n}(x_i^2+x_{i+1}^2) \leq \max_{j=1,\ldots,n}(2x_jx_{j+1}),$$

where $x_{n+1}=x_1$.

Problem 2. Let $ABCD$ be a cyclic quadrilateral, and let diagonals AC and BD intersect at X. Let C_1, D_1 and M be the midpoints of segments CX, DX and CD, respectively. Lines AD_1 and BC_1 intersect at Y, and line MY intersects diagonals AC and BD at different points E and F, respectively. Prove that line XY is tangent to the circle through E, F and X.

Problem 3. Let m be a positive integer. Consider a $4m \times 4m$ array of square unit cells. Two different cells are *related* to each other if they are in either the same row or in the same column. No cell is related to itself. Some cells are coloured blue, such that every cell is related to at least two blue cells. Determine the minimum number of blue cells.

Time: 4 hours and 30 minutes
Each problem is worth 7 points