Problem 1. Let $n$ be an odd positive integer, and let $x_1, x_2, \ldots, x_n$ be non-negative real numbers. Show that
\[
\min_{i=1,\ldots,n} (x_i^2 + x_{i+1}^2) \leq \max_{j=1,\ldots,n} (2x_j x_{j+1}),
\]
where $x_{n+1} = x_1$.

Problem 2. Let $ABCD$ be a cyclic quadrilateral, and let diagonals $AC$ and $BD$ intersect at $X$. Let $C_1, D_1$ and $M$ be the midpoints of segments $CX$, $DX$ and $CD$, respectively. Lines $AD_1$ and $BC_1$ intersect at $Y$, and line $MY$ intersects diagonals $AC$ and $BD$ at different points $E$ and $F$, respectively. Prove that line $XY$ is tangent to the circle through $E$, $F$ and $X$.

Problem 3. Let $m$ be a positive integer. Consider a $4m \times 4m$ array of square unit cells. Two different cells are related to each other if they are in either the same row or in the same column. No cell is related to itself. Some cells are coloured blue, such that every cell is related to at least two blue cells. Determine the minimum number of blue cells.