Language: **English**

Day: **2**

Friday, April 17, 2015

Problem 4. Determine whether there exists an infinite sequence $a_1, a_2, a_3, \ldots$ of positive integers which satisfies the equality

$$a_{n+2} = a_{n+1} + \sqrt{a_{n+1} + a_n}$$

for every positive integer n.

Problem 5. Let m, n be positive integers with $m > 1$. Anastasia partitions the integers $1, 2, \ldots, 2m$ into m pairs. Boris then chooses one integer from each pair and finds the sum of these chosen integers. Prove that Anastasia can select the pairs so that Boris cannot make his sum equal to n.

Problem 6. Let H be the orthocentre and G be the centroid of acute-angled triangle $\triangle ABC$ with $AB \neq AC$. The line AG intersects the circumcircle of $\triangle ABC$ at A and P. Let P' be the reflection of P in the line BC. Prove that $\angle CAB = 60^\circ$ if and only if $HG = GP'$.

Time: 4 hours and 30 minutes
Each problem is worth 7 points