Problem 1. Let $\triangle ABC$ be an acute-angled triangle, and let $D$ be the foot of the altitude from $C$. The angle bisector of $\angle ABC$ intersects $CD$ at $E$ and meets the circumcircle $\omega$ of triangle $\triangle ADE$ again at $F$. If $\angle ADF = 45^\circ$, show that $CF$ is tangent to $\omega$.

Problem 2. A domino is a $2 \times 1$ or $1 \times 2$ tile. Determine in how many ways exactly $n^2$ dominoes can be placed without overlapping on a $2n \times 2n$ chessboard so that every $2 \times 2$ square contains at least two uncovered unit squares which lie in the same row or column.

Problem 3. Let $n, m$ be integers greater than 1, and let $a_1, a_2, \ldots, a_m$ be positive integers not greater than $n^m$. Prove that there exist positive integers $b_1, b_2, \ldots, b_m$ not greater than $n$, such that

$$\gcd(a_1 + b_1, a_2 + b_2, \ldots, a_m + b_m) < n,$$

where $\gcd(x_1, x_2, \ldots, x_m)$ denotes the greatest common divisor of $x_1, x_2, \ldots, x_m$. 