Language: **English**

Day: **2**

*Sunday, April 13, 2014*

**Problem 4.** Determine all integers $n \geq 2$ for which there exist integers $x_1, x_2, \ldots, x_{n-1}$ satisfying the condition that if $0 < i < n$, $0 < j < n$, $i \neq j$ and $n$ divides $2i + j$, then $x_i < x_j$.

**Problem 5.** Let $n$ be a positive integer. We have $n$ boxes where each box contains a non-negative number of pebbles. In each move we are allowed to take two pebbles from a box we choose, throw away one of the pebbles and put the other pebble in another box we choose. An initial configuration of pebbles is called *solvable* if it is possible to reach a configuration with no empty box, in a finite (possibly zero) number of moves. Determine all initial configurations of pebbles which are not solvable, but become solvable when an additional pebble is added to a box, no matter which box is chosen.

**Problem 6.** Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying the condition

$$f(y^2 + 2xf(y) + f(x)^2) = (y + f(x))(x + f(y))$$

for all real numbers $x$ and $y$.

Language: English

Time: 4 hours and 30 minutes
Each problem is worth 7 points