Saturday, April 12, 2014

Problem 1. Determine all real constants t such that whenever a, b, c are the lengths of the sides of a triangle, then so are $a^2 + bct$, $b^2 + cat$, $c^2 + abt$.

Problem 2. Let D and E be points in the interiors of sides AB and AC, respectively, of a triangle ABC, such that $DB = BC = CE$. Let the lines CD and BE meet at F. Prove that the incentre I of triangle ABC, the orthocentre H of triangle DEF and the midpoint M of the arc BAC of the circumcircle of triangle ABC are collinear.

Problem 3. We denote the number of positive divisors of a positive integer m by $d(m)$ and the number of distinct prime divisors of m by $\omega(m)$. Let k be a positive integer. Prove that there exist infinitely many positive integers n such that $\omega(n) = k$ and $d(n)$ does not divide $d(a^2 + b^2)$ for any positive integers a, b satisfying $a + b = n$.

Time: 4 hours and 30 minutes
Each problem is worth 7 points