

EGMO 2013
European Girls' Mathematical Olympiad

Problem 4. Find all positive integers $a$ and $b$ for which there are three consecutive integers at which the polynomial

$$
P(n)=\frac{n^{5}+a}{b}
$$

takes integer values.

Problem 5. Let $\Omega$ be the circumcircle of the triangle $A B C$. The circle $\omega$ is tangent to the sides $A C$ and $B C$, and it is internally tangent to the circle $\Omega$ at the point $P$. A line parallel to $A B$ and intersecting the interior of triangle $A B C$ is tangent to $\omega$ at $Q$.

Prove that $\angle A C P=\angle Q C B$.

Problem 6. Snow White and the Seven Dwarves are living in their house in the forest. On each of 16 consecutive days, some of the dwarves worked in the diamond mine while the remaining dwarves collected berries in the forest. No dwarf performed both types of work on the same day. On any two different (not necessarily consecutive) days, at least three dwarves each performed both types of work. Further, on the first day, all seven dwarves worked in the diamond mine.

Prove that, on one of these 16 days, all seven dwarves were collecting berries.

