Language: **English**

Day: 2

Sunday, April 12, 2026

Problem 4. Let $1 = a_1 \geq a_2 \geq a_3 \geq \cdots$ be an infinite sequence of real numbers such that $a_n = a_{2n} + a_{2n+1}$ for all positive integers n. For $r = 2026^{2026}$, prove that

$$\frac{1}{r} \leq a_r \leq \frac{2}{r+1}.$$

Problem 5. Let ABC be an acute triangle with $AC > AB$. Denote by ω its circumcircle and by O its circumcentre. Let K be the intersection of the tangents to ω at B and C. Circle ABK intersects line BC again at $Z \neq B$. Let L be the midpoint of KZ. Let X be the intersection of lines KZ and AB. Let V be the point on circle ABL on the same side of BC as A such that OV is perpendicular to KZ. Prove that LV is perpendicular to CX.

Problem 6. Let p be a prime number and let n be a positive integer such that p does **not** divide n. Denote by k the number of positive divisors of n, and by $1 = d_1 < d_2 < \cdots < d_k = n$ the positive divisors of n. For $i = 1, 2, \ldots, k$, let c_i be the number of positive divisors ℓ of d_i^2 such that $d_i - \ell$ is divisible by p. Prove that

$$(p-1)(c_1 + c_2 + \cdots + c_k) \geq k^2.$$

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Time: 4 hours and 30 minutes
Each problem is worth 7 points