



*Kuwa Gatandatu, 11 Mata, 2026*

**Ikibazo cya 1.** Kare (square) ya  $2026 \times 2026$  yitwa *bordeaux* iyo byibura kamwe mu tuzu duto (cells) tuyigize ari umutuku. Hanyuma rectangle yitwa *oddly-rectangular* iyo mu tuzu duto tuyigize udusa umutuku ari igiharwe (odd number). Shaka umubare munini ushoboka  $M$  kuburyo buri gihe muri ya kare nini ya  $2026 \times 2026$  tuzabonamo *oddly-rectangular* ifite byibuze utuzu duto  $M$ .

*Ikitonderwa:* Muri rectangle ziri kuvugwa zose ni aho impande zayo ziringaniye n'impande za kare nini ya  $2026 \times 2026$ , ikaba igizwe n'utuzu duto twose turimo imbere.

**Ikibazo cya 2.** Reka  $n$  ibe positive integer. Mariya akina umukino ahereye kuri 1 yanditse kuri blackboard. Umukino uteye gutya: Mariya atekereza umubare  $j$ , uri hagati ya rimwe na  $n$  nabyo birimo ( $1 \leq j \leq n$ ). Hanyuma ashaka umubare mushya  $j \cdot R\left(\frac{V}{j}\right)$  akawusimbuzwa umubare  $V$  wari wanditse kuri blackboard. Ibyo bintu abikora inshoro nyinshi ashaka.  $R(x)$  ni fonkisiyo itanga umubare ushyitse (integer) wegereye  $x$ ; iyo uwo mubare uri hagati na hagati, ujya kumubare uri hejuru. Ingero:  $R(1.3) = 1$ ,  $R(1.5) = R(1.8) = 2$ .

- Erekana ko buri mubare  $n$  wafata, hari umubare  $B$  kuburyo Mariya adashobora kubona umubare uruta  $B$  kuri blackboard.
- Kuri buri mubare  $n$ , reka  $f(n)$  ibe fonkisiyo itanga umubare munini ishoboka kuri blackboard nyuma yuko Mariya ikinye inshuro runaka. Erekana ko hari positive integer  $N$ , ku buryo buri mibare yose  $n \geq N$ , tuzaba dufite ko 2026 igabanya  $f(n)$ .

**Ikibazo cya 3.** Reka  $\mathbb{R}$  ibe real numbers zose. Shaka functions zose  $f: \mathbb{R} \rightarrow \mathbb{R}$  ya real numbers  $x, y$  ku buryo iyi ekwaziyo ari ukuri:

$$f((f(x) + f(y))^2) = (x + y)f(x + y).$$