Language: **English**

Day: **1**

Saturday, April 15, 2023

Problem 1. There are $n \geqslant 3$ positive real numbers $a_1, a_2, \ldots, a_n$. For each $1 \leqslant i \leqslant n$ we let $b_i = \frac{a_{i-1}+a_{i+1}}{a_i}$ (here we define a_0 to be a_n and a_{n+1} to be a_1). Assume that for all i and j in the range 1 to n, we have $a_i \leqslant a_j$ if and only if $b_i \leqslant b_j$.

Prove that $a_1 = a_2 = \cdots = a_n$.

Problem 2. We are given an acute triangle ABC. Let D be the point on its circumcircle such that AD is a diameter. Suppose that points K and L lie on segments AB and AC, respectively, and that DK and DL are tangent to circle AKL.

Show that line KL passes through the orthocentre of ABC.

The orthocentre of a triangle is the point of intersection of its altitudes.

Problem 3. Let k be a positive integer. Lexi has a dictionary $\mathcal{D}$ consisting of some k-letter strings containing only the letters A and B. Lexi would like to write either the letter A or the letter B in each cell of a $k \times k$ grid so that each column contains a string from $\mathcal{D}$ when read from top-to-bottom and each row contains a string from $\mathcal{D}$ when read from left-to-right.

What is the smallest integer m such that if $\mathcal{D}$ contains at least m different strings, then Lexi can fill her grid in this manner, no matter what strings are in $\mathcal{D}$?

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

The problems are confidential until Sunday 16 April, 22:00 UTC (00:00 (Monday) Central European Summer Time).