Problem 4. Given a positive integer \( n \geq 2 \), determine the largest positive integer \( N \) for which there exist \( N + 1 \) real numbers \( a_0, a_1, \ldots, a_N \) such that

\[
(1) \quad a_0 + a_1 = -\frac{1}{n}, \text{ and}
\]

\[
(2) \quad (a_k + a_{k-1})(a_k + a_{k+1}) = a_{k-1} - a_{k+1} \quad \text{for} \quad 1 \leq k \leq N - 1.
\]

Problem 5. For all positive integers \( n, k \), let \( f(n, 2k) \) be the number of ways an \( n \times 2k \) board can be fully covered by \( nk \) dominoes of size \( 2 \times 1 \). (For example, \( f(2, 2) = 2 \) and \( f(3, 2) = 3 \).) Find all positive integers \( n \) such that for every positive integer \( k \), the number \( f(n, 2k) \) is odd.

Problem 6. Let \( ABCD \) be a cyclic quadrilateral with circumcentre \( O \). Let the internal angle bisectors at \( A \) and \( B \) meet at \( X \), the internal angle bisectors at \( B \) and \( C \) meet at \( Y \), the internal angle bisectors at \( C \) and \( D \) meet at \( Z \), and the internal angle bisectors at \( D \) and \( A \) meet at \( W \). Further, let \( AC \) and \( BD \) meet at \( P \). Suppose that the points \( X, Y, Z, W, O \) and \( P \) are distinct.

Prove that \( O, X, Y, Z \) and \( W \) lie on the same circle if and only if \( P, X, Y, Z \) and \( W \) lie on the same circle.

Language: English

To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Saturday 9 April, 22:00 UTC (15:00 Pacific Daylight Time, 00:00 (Sunday) Central European Summer Time, 08:00 (Sunday) Australian Eastern Standard Time).