EGMO 2021
GEORGIA
KUTAISI

Language: **English (US)**

Day: **1**

Sunday, April 11, 2021

Problem 1. The number 2021 is *fantabulous*. For any positive integer m, if any element of the set $\{m, 2m+1, 3m\}$ is fantabulous, then all the elements are fantabulous. Does it follow that the number 2021^{2021} is fantabulous?

Problem 2. Find all functions $f\colon \mathbb{Q} \to \mathbb{Q}$ such that the equation

$$f(xf(x)+y) = f(y) + x^2$$

holds for all rational numbers x and y.

Here, $\mathbb{Q}$ denotes the set of rational numbers.

Problem 3. Let ABC be a triangle with an obtuse angle at A. Let E and F be the intersections of the external bisector of angle A with the altitudes of ABC through B and C respectively. Let M and N be the points on the segments EC and FB respectively such that $\angle EMA = \angle BCA$ and $\angle ANF = \angle ABC$. Prove that the points E, F, N, M lie on a circle.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Tuesday 13 April, 12:00 UTC (05:00 Pacific Daylight Time, 13:00 British Summer Time, 22:00 Australian Eastern Standard Time).